

PHYS 451 Quantum Mechanics II (Fall 2018)
Quiz #3

Consider a two-level system with the following Hamiltonian (in some appropriate units):

$$H = \begin{pmatrix} 2 & i \\ -i & 2 \end{pmatrix}.$$

The system is subject to a weak additional interaction in the form

$$V = \gamma \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix},$$

where $\gamma \ll 1$. Use the perturbation theory to compute the energy levels up to the second order in γ .

Appendix: Perturbation theory formulae (from lecture)

$$H = H^0 + \lambda H', \quad E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \dots, \quad \psi_n = \psi_n^{(0)} + \lambda \psi_n^{(1)} + \lambda^2 \psi_n^{(2)} + \dots$$

$$E_n^{(1)} = H'_{nn}$$

$$\psi_n^{(1)} = \sum_m c_{nm} \psi_m^{(0)}, \quad c_{nm} = \begin{cases} \frac{H'_{mn}}{E_n^{(0)} - E_m^{(0)}}, & n \neq m \\ 0, & n = m \end{cases}$$

$$E_n^{(2)} = \sum_{m \neq n} \frac{|H'_{mn}|^2}{E_n^{(0)} - E_m^{(0)}}$$

$$\psi_n^{(2)} = \sum_m d_{nm} \psi_m^{(0)}, \quad d_{nm} = \begin{cases} \frac{1}{E_n^{(0)} - E_m^{(0)}} \left(\sum_{k \neq n} \frac{H'_{mk} H'_{kn}}{E_n^{(0)} - E_k^{(0)}} \right) - \frac{H'_{nn} H'_{mn}}{(E_n^{(0)} - E_m^{(0)})^2}, & n \neq m \\ 0, & n = m \end{cases}$$