PHYS 452 Quantum Mechanics II (Fall 2019)
Homework #2, due Tuesday Sept 10 in class

Perturbation theory

1. A particle of mass m moving in an infinite potential well of width a $(0 < x < a)$ is subject to a small perturbative potential $V(x) = V_0 \cos^2 \frac{\pi x}{a}$. Find the shifts of the energy levels up to the second order in V.

2. Consider a weakly relativistic particle of mass m that moves in the potential $V(x) = \frac{1}{2} m\omega^2 x^2$. Begin with the relativistic formula for the total energy, $E^2 = m^2c^4 + p^2c^2$. Then, assuming that the magnitude of the particle's momentum is much smaller than mc, expand the expression for the kinetic energy in powers of p/mc. As a result, after dropping higher order terms, you should obtain the Hamiltonian that is the sum of the ordinary nonrelativistic Hamiltonian for the harmonic oscillator plus some small perturbation. Note that the perturbation here will contain the momentum operator. Lastly, apply the perturbation theory to find the ground state energy accurate up to order $1/c^2$.

3. Consider a three-level system with the Hamiltonian

$$H = \epsilon \begin{pmatrix} 1 & 0 & \alpha \\ 0 & 2+\alpha & 0 \\ \alpha & 0 & 3 \end{pmatrix},$$

   where ϵ is a constant that has units of energy and α is a small dimensionless parameter. Find corrections to the energy levels and wave functions up to the lowest non-vanishing order in α.

4. Consider a particle of mass m moving in the potential that has the following form

$$V(x) = -V_0 e^{-\beta x^2},$$

   where V_0 and β are positive constants. The exact solution to the Schrödinger equation with this potential is not known. Under certain condition, however, the ground state energy level is lies deep at the bottom of the well and the problem can be treated perturbatively.

   (a) What is this condition?

   (b) Find the expansion of the ground state energy in powers of the relevant small parameter up to the lowest nontrivial order.

Found an error or need a clarification? Email the instructor at sergiy.bubin@nu.edu.kz