PHYS 452 Quantum Mechanics II (Fall 2019)
Homework #3, due Thursday Sep 19 in class

Perturbation theory, Stark effect, Hyperfine structure

1. Consider a free electron in a magnetic field that is a combination of two uniform magnetic fields. The first one is along the z-direction, $\mathbf{B}_1 = (0, 0, B_z)$. The second one is along the x-direction, $\mathbf{B}_2 = (B_x, 0, 0)$.

   (a) Assuming $B_x \ll B_z$ use the perturbation theory to find the energies and eigenstates up to the lowest non-vanishing order.

   (b) Solve the problem exactly and see if the perturbative solution in the previous part reproduces the exact solution when $B_x \ll B_z$.

2. Consider the *second excited* state of a quantum harmonic oscillator in 2D. The Hamiltonian of this system is given by

$$H = \frac{(p_x^2 + p_y^2)}{2m} + \frac{m\omega^2(x^2 + y^2)}{2}.$$

   Suppose this system is subjected to a weak perturbation in the form $H' = \alpha xy$, where $\alpha \ll m\omega^2$. Find the first-order corrections to the energy levels and the new eigenfunctions in terms of the unperturbed states.

3. Problem 6.40a (part b is not required) in Griffiths.

4. Consider the ground state of the hydrogen atom, consisting of an electron and proton. Due to the spins of the electron and proton, the ground state is four-times degenerate, i.e. we have four possible spin states $|{\uparrow_e \uparrow_p}\rangle$, $|{\uparrow_e \downarrow_p}\rangle$, $|{\downarrow_e \uparrow_p}\rangle$, and $|{\downarrow_e \downarrow_p}\rangle$. The atom is placed in the uniform magnetic field $\mathbf{B} = (0, 0, B)$. The effective Hamiltonian for the hyperfine structure in the presence of this field can be written as

$$H' = \frac{\kappa}{\hbar^2}\mathbf{S}\cdot\mathbf{I} + \frac{\beta}{\hbar}S_z - \frac{\gamma}{\hbar}I_z,$$

   where κ, β, and γ are some positive constants, $\mathbf{S}$ is the electron spin, and $\mathbf{I}$ is the proton spin. What are the first-order corrections to the energy of the ground state due to H'? Hint: you might want to express $\mathbf{S}\cdot\mathbf{I}$ through raising and lowering operators $S_\pm$ and $I_\pm$.