StudentID: _____

PHYS 452 Quantum Mechanics II (Fall 2019)
Quiz #3

A spin 1/2 particle with gyromagnetic ratio γ (the ratio of the particle's magnetic moment μ to its spin angular momentum S) is placed in a uniform magnetic field of magnitude B that is directed along the y-axis.

(a) What is the Hamiltonian of this system (in the basis of the eigenstates of S_z)?

(b) What are the possible energy levels of the particle and the corresponding eigenstates?

(c) Now a weak perpendicular field is added. This weak field points in the z-direction and its magnitude αB is a small fraction of the original field B (here $\alpha \ll 1$). Using the perturbation theory find the corrections to the energy levels up to the lowest nonvanishing order.

Appendix 1: Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Appendix 2: Perturbation theory formulae (from lecture)

$$H = H^0 + \lambda H', \qquad E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \dots\ , \qquad \psi_n = \psi_n^{(0)} + \lambda \psi_n^{(1)} + \lambda^2 \psi_n^{(2)} + \dots$$

$$E_n^{(1)} = H'_{nn}$$

$$\psi_n^{(1)} = \sum_m c_{nm} \psi_m^{(0)}, \quad c_{nm} = \begin{cases} \frac{H'_{mn}}{E_n^{(0)} - E_m^{(0)}}, & n \neq m \\ 0, & n = m \end{cases}$$

$$E_n^{(2)} = \sum_{m \neq n} \frac{|H'_{mn}|^2}{E_n^{(0)} - E_m^{(0)}}$$

$$\psi_n^{(2)} = \sum_m d_{nm} \psi_m^{(0)}, \quad d_{nm} = \begin{cases} \frac{1}{E_n^{(0)} - E_m^{(0)}} \left(\sum_{k \neq n} \frac{H'_{mk} H'_{kn}}{E_n^{(0)} - E_k^{(0)}} \right) - \frac{H'_{nn} H'_{mn}}{\left(E_n^{(0)} - E_m^{(0)} \right)^2}, & n \neq m \\ 0, & n = m \end{cases}$$