StudentID: ____________________

PHYS 452 Quantum Mechanics II (Fall 2019)
Quiz #4

In a recent lecture we considered a two-level system with the Hamiltonian $H(t) = H^0 + V(t)$. We formally expanded the general solution as $\Psi(x,t) = \sum_{k=1}^{2} c_k(t)\psi_k(x)e^{-i\omega_k t}$ (here $\omega_k \equiv E_k/\hbar$ and ψ_k are the solutions to the stationary Schrödinger equations, $H^0\psi_k = E_k\psi_k$) and obtained the time-dependent Schrödinger equation in the matrix form:

$$\begin{pmatrix} \mathsf{V}_{11} & \mathsf{V}_{12}e^{-i\omega_{21}t} \\ \mathsf{V}_{21}e^{i\omega_{21}t} & \mathsf{V}_{22} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = i\hbar \begin{pmatrix} \dot{c}_1 \\ \dot{c}_2 \end{pmatrix} \qquad \mathsf{V}_{km}(t) \equiv \langle\psi_k|V(t)|\psi_m\rangle \qquad \omega_{21} \equiv \omega_2 - \omega_1\,.$$

Now do a similar exercise – expand formally the general solution as $\Psi(x,t) = \sum_{k=1}^{2} a_k(t)\phi_k(x)$, where $a_k(t)$ are time-dependent expansion coefficients and $\phi_k(x)$ are some arbitrary time-independent basis states that are not necessarily orthogonal to each other. Rewrite the time-dependent Schrödinger equation in the matrix form. You may want to denote $\mathsf{H}_{km}(t) \equiv \langle\phi_k|H(t)|\phi_m\rangle$ and $\mathsf{S}_{km} \equiv \langle\phi_k|\phi_m\rangle$.