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PHYS 452 Quantum Mechanics II (Fall 2019)
Quiz #5

Consider a particle of mass m in the ground state ψ_1 of the 1D infinite square well ($0 \le x \le a$). At time $t = 0$ the potential is changed to

$$V(x) = \begin{cases} V_0, & 0 \le x \le \frac{a}{2} \\ 0, & \frac{a}{2} < x \le a \\ \infty, & \text{otherwise} \end{cases},$$

and at time $t = T$ the potential is reverted back to the original 1D infinite square well. Find the probability that the particle makes a transition to the first excited state ψ_2.

Appendix:

Infinite square well ($0 \le x \le a$)
Energy levels: $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$, $n = 1, 2, ..., \infty$
Eigenfunctions: $\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right) \quad (0 \le x \le a)$

Time-dependence of the wave function

$H(\mathbf{r}, t) = H^0(\mathbf{r}) + \lambda H'(\mathbf{r}, t), \qquad H^0\phi_n = E_n\phi_n, \qquad \Psi(\mathbf{r}, t) = \sum_n c_n(t)\phi_n(\mathbf{r})e^{\frac{-iE_nt}{\hbar}},$

$i\hbar\frac{dc_n(t)}{dt} = \lambda\sum_k H'_{nk}e^{i\omega_{nk}t}c_k(t), \qquad H'_{nk}(t) = \langle\phi_n|H'|\phi_k\rangle, \qquad \omega_{nk} = \frac{E_n - E_k}{\hbar}$

Time-dependent perturbation theory formulae

If $c_n(t_0) = \delta_{nm}$ (e.g. $\Psi(\mathbf{r}, t_0) = \phi_m(\mathbf{r})$) and $\lambda H'$ is small then at time $t > t_0$
$c_n(t) = c_n^{(0)} + \lambda c_n^{(1)} + \lambda^2 c_n^{(2)} + \dots$
where
$c_n^{(0)} = \delta_{nm},$
$c_n^{(1)}(t) = \frac{1}{i\hbar}\int_{t_0}^{t} H'_{nm}(t')\, e^{i\omega_{nm}t'} dt',$
$c_n^{(2)}(t) = \left(\frac{1}{i\hbar}\right)^2 \sum_k \int_{t_0}^{t} dt' \int_{t_0}^{t'} H'_{nk}(t')H'_{km}(t'')\, e^{i\omega_{nk}t'}e^{i\omega_{km}t''} dt'', \quad \dots$

Useful trigonometric identities

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \qquad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)] \qquad \cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$
$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)] \qquad \cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$