Name: ____________________

PHYS 505 - Classical Mechanics (graduate), Fall 2015
Instructor: Sergiy Bubin
Diagnostic Test

Instructions:

- The purpose of this test is to determine the level of student's knowledge of mechanics and their basic mathematical skills. No test score will be issued.
- This is a closed book test. No notes, books, phones, tablets, calculators, etc. should be used.
- No communication with classmates is allowed during the test.
- Show your work, explain your reasoning.
- Write legibly and make sure pages are stapled together before submitting your work.

Problem 1. Given the following function of two variables x and y

$$f(x, y) = \sin(x + y)$$

find its Maclaurin series up to the terms of the third order.

Problem 2. Solve the following equation (β and γ are constants):

$$\frac{d^2 f(t)}{dt^2} + \beta \frac{df(t)}{dt} + \gamma f(t) = 0$$

given that $f(0) = a$ and $f'(0) = 0$.

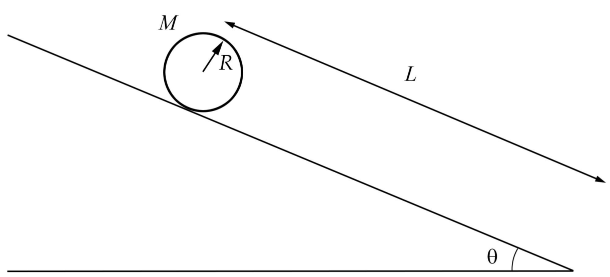
Problem 3. If $\psi(\mathbf{r})$ and $\mathbf{A}(\mathbf{r})$, where $\mathbf{r} = (x, y, z)$, are some differentiable scalar and vector functions respectively, simplify the following expressions

(a) $\nabla \cdot (\psi \mathbf{A})$

(b) $\nabla \cdot (\nabla \times \psi \mathbf{A})$

(c) $\nabla^2 (\psi \mathbf{A})$

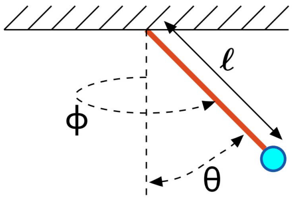
Problem 4. A uniform cylinder of mass M and radius R rolls down a ramp without slipping. The ramp makes angle θ with the horizontal surface. What is the velocity of the cylinder at the bottom of the ramp?



Problem 5. Consider a free particle, whose kinetic energy is small compared to its rest mass. Using the relativistic relation between the energy, momentum and rest mass, derive the leading correction to the nonrelativistic kinetic energy, $\frac{p^2}{2m}$.

Problem 6. Consider a classical harmonic oscillator of angular frequency ω (e.g. a particle of mass m subjected to force $F(x) = -kx$, where k is a constant) that oscillates along coordinate x with amplitude A. If we measure the position at a random time, find the probability distribution $\rho(x)$ that the particle would be found in the infinitesimal interval $[x, x + dx]$.

Problem 7. A point mass m is attached to the ceiling by a massless string of length l. The position of this point mass is then determined by two angles, ϕ and θ.



(a) Write down the expressions for the kinetic and potential energies in terms of m, g, l, ϕ, θ, $\dot{\phi}$, and $\dot{\theta}$.

(b) Using the Lagrangian method, derive equations of motion for both ϕ and θ?

(c) Find an expression for the angular momentum L_z in terms of m, g, l, ϕ, θ, $\dot{\phi}$, and $\dot{\theta}$.

(d) Rewrite the equations of motion for θ so that any mention of $\dot{\phi}$ is replaced by L_z.

(e) Consider a circular trajectory of constant $\theta = \pi/2$. What is L_z and what is the frequency of small oscillations of θ about $\pi/2$? Give answers in terms of m, g, l.