PHYS 505: Classical Mechanics (graduate) - Fall 2015
Homework #1, due Friday September 25, in class

Review of elementary vector calculus and Newtonian mechanics

1. Compute $\nabla^2(e^{-\alpha|\mathbf{r}|})$ (where $\mathbf{r} \equiv (x, y, z)$).
2. Is there a vector field $\mathbf{F}$ such that $\nabla \times \mathbf{F} = (x + y, z, y^2)$. If so, find an expression for it.
3. The temperature at point (x, y, z) is given by $T(x, y, z) = 273 + e^{-(10-x)^2} + e^{-y^2} + e^{-z^2}$. A bird flies along the path $\mathbf{r}(t) = (t^3,\, 2\sin(t),\, e^{-t} - t)$. The units for the temperature, distance, and time are Kelvin, meter, and second, respectively. Find the rate of change of temperature experienced by the bird at the moment of time $t = 0$.
4. A particle travels along a straight line from point $(0, 0, 0)$ to point $(1, 0, 0)$, then to point $(1, 2, 1)$, then to point $(0, 2, 1)$, and then gets back to the origin. During its travel the particle is subjected to the force field $\mathbf{F} = (z^2, 2xy, 4y^2)$. Find the work done in two separate ways: (a) by directly calculating a trajectory integral, and (b) by using Stokes' theorem with a suitable choice of surface S.
5. Prove that for a particle moving with velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ the following is true: $|\mathbf{v} \times \mathbf{a}| = v^3/\rho$, where ρ is the radius of curvature of the trajectory.
6. A ball of mass m is taken on a helicopter to a very high altitude and dropped. Find the velocity as a function of time if the air resistance is proportional to the velocity of the ball, e.g. $F(v) = -\beta v$. With what speed the ball is going to hit the surface? Assume there is no wind in the air.
7. Imagine that you could dig a straight tunnel that goes through the center of the Earth. What would be the period of small oscillations of a ball inside that tunnel? What would be the travel time from surface to surface? Assume that the tunnel is filled with vacuum and there is no air resistance.
8. Consider a cone of height h and mass m that is suspended from the ceiling with a weightless rope of length l (the rope is attached to the sharp end of the cone). Find the period of small oscillations of this system around the equilibrium position.
9. Problem 1.1 in Goldstein.
10. Problem 1.3 in Goldstein.