**PHYS 505: Classical Mechanics (graduate) - Fall 2015**
**Homework #3, due Friday October 9, in class**

Review of basic math. The calculus of variations. Constrained motion.

1. Find the Taylor expansion of the following functions of one and several variables:

   (a) $f(x) = e^x \sin(x)$ at $x = 0$, up to the fourth order.

   (b) $f(x) = \int_0^x \arctan(t)dt$ at $x = 0$, up to the third order.

   (c) $f(x) = g''(x)$, where $g(x) = x^3 \exp(-x^2)$, at $x = 0$, up to the seventh order.

   (d) $f(x) = \Gamma(x)$ at $x = 1$ up to the first order. Here $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$ is the Euler Gamma function.

   (e) $f(x, y) = \frac{e^{(x+y)^2} \ln(1+y)}{1-x}$ at $x = y = 0$, up to the second order.

   (f) $f(\mathbf{r}) = \frac{1}{|\mathbf{a}-\mathbf{r}|}$ at $\mathbf{r} = (0, 0, 0)$, up to the second order.

2. Find the maximum volume of a lidless rectangular box made from 12 m$^2$ of plywood.

3. Find the curve $y(x)$ of fixed length $l$ that passes through points (0,0) and (1,0) and which maximizes the area between the curve and the $x$-axis.

4. Problem 2.12 in Goldstein.

5. Problem 2.20 in Goldstein.