Rotational motion of a rigid body.

1. Find the principal moments of inertia of a solid hemisphere of radius $R$ about its center of mass. Assume that the hemisphere has uniform density $\rho$.

2. What is the kinetic energy of a thin uniform square plate of side $a$ and mass $m$ when it is rotated about its diagonal with angular velocity $\omega$?

3. It is customary to use Euler’s angles $(\phi, \theta, \psi)$ to parametrise an arbitrary rotation of a rigid body. However, there exist other convenient choices. One of them is to specify a rotation axis (given by a unit vector, $\mathbf{n}$) and the angle of rotation about this axis, $\Phi$. Find the connection between the two sets of parameters.

4. A thin uniform disk of radius $R$ and mass $m$ is rigidly attached (through its center of mass) to an axle and tilted as shown in the figure below. The normal to the disk makes an angle $\theta$ with the axle. The axle rotates with angular velocity $\omega$.

   ![Diagram of a disk tilted on an axle](image)

   (a) Find the principal moments of inertia of the disk.

   (b) The disk is unbalanced. By considering the Euler equations find the components of the torque vector that maintains its motion.

5. How fast must a 100 tenge coin (which is 25 mm in diameter) be rolling on the floor in order to remain upright?

Found an error or need a clarification? Email the instructor at sergiy.bubin@nu.edu.kz