1. In lecture we derived Green’s function for an ideal forced harmonic oscillator. Based on the general approach we used in class, derive the Green function for a damped forced harmonic oscillator described by the equation

\[ m\ddot{x} + b\dot{x} + kx = F(t). \]

Use that Green function to find \( x(t) \) in the limit when the damping is weak, \( t \) is very large, and the forcing function is given by

\[ F(t) = \begin{cases} \alpha t, & t \geq 0 \\ 0, & t < 0 \end{cases} \]

2. Consider a hypothetical diatomic molecule in which the potential energy of the two ions is

\[ V(r) = -\frac{\alpha}{r} + \frac{\beta}{r^9}. \]

Here the first term is the Coulomb interaction, while the second term accounts for the repulsion of the two ions at small distance. Find \( \beta \) as a function of the equilibrium bond length, \( r_e \). What is the frequency of small oscillations about \( r = r_e \)? Assume that the reduced mass of the two ions is \( \mu \).

3. Consider a simple model for the vibration of a linear triatomic molecule where the atom in the center has mass \( M \), while the other two have masses \( m \) (e.g. BeH\(_2\) molecule). The three atoms are connected with two identical springs of force constant \( k \).

(a) Calculate the eigenfrequencies of such a molecule.
(b) Discuss the vibration modes (i.e. indicate what moves in what direction).

4. The double pendulum shown in the figure below consists of two simple pendula of mass \( m \) and length \( l \). The upper one is fixed to the ceiling, while the second one is attached to the mass of the first. The motion is restricted to the \( xy \) plane. Find the normal modes and the corresponding normal mode frequencies.

Found an error or need a clarification? Email the instructor at sergiy.bubin@nu.edu.kz