

**PHYS 505: Classical Mechanics (graduate) - Fall 2015**  
**Homework #6, due Friday November 20, in class**

Oscillations.

1. In lecture we derived Green's function for an ideal forced harmonic oscillator. Based on the general approach we used in class, derive the Green function for a *damped* forced harmonic oscillator described by the equation

$$m\ddot{x} + b\dot{x} + kx = F(t).$$

Use that Green function to find  $x(t)$  in the limit when the damping is weak,  $t$  is very large, and the forcing function is given by

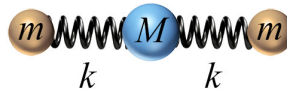
$$F(t) = \begin{cases} \alpha t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

2. Consider a hypothetical diatomic molecule in which the potential energy of the two ions is

$$V(r) = -\frac{\alpha}{r} + \frac{\beta}{r^9}.$$

Here the first term is the Coulomb interaction, while the second term accounts for the repulsion of the two ions at small distance. Find  $\beta$  as a function of the equilibrium bond length,  $r_e$ . What is the frequency of small oscillations about  $r = r_e$ ? Assume that the reduced mass of the two ions is  $\mu$ .

3. Consider a simple model for the vibration of a linear triatomic molecule where the atom in the center has mass  $M$ , while the other two have masses  $m$  (e.g.  $\text{BeH}_2$  molecule). The three atoms are connected with two identical springs of force constant  $k$ .



- (a) Calculate the eigenfrequencies of such a molecule.  
 (b) Discuss the vibration modes (i.e. indicate what moves in what direction).
4. The double pendulum shown in the figure below consists of two simple pendula of mass  $m$  and length  $l$ . The upper one is fixed to the ceiling, while the second one is attached to the mass of the first. The motion is restricted to the  $xy$  plane. Find the normal modes and the corresponding normal mode frequencies.

