Problem 1. Consider the system of three masses connected with two weightless pulleys as shown in the figure. The top pulley’s position is fixed, while the bottom one can move up and down. Write down the Lagrangian for this system, generate Lagrange equation(s), identify any cyclic coordinate(s), and solve the equation(s). What are the accelerations of masses $m_1$, $m_2$, and $m_3$?

Problem 2. A block of mass $M$ is free to slide forth and back in the $x$-direction on a horizontal bar without friction. A bob of mass $m$ is attached to the bottom of the block with a massless rigid rod of length $l$. The bob can oscillate freely in the $xy$-plane. Write down the Lagrangian for this system, generate Lagrange equation(s), identify any cyclic coordinate(s), and solve the equation(s) in the case of small amplitude oscillations.
Problem 3. The Fermat principle states that the light propagates along the path which takes the minimum amount of time. Recall that the local speed of light in a medium with the index of refraction $n$ is $v = c/n$, where $c$ is the speed of light in vacuum. From the Fermat principle it follows that for a medium with a nonuniform index of refraction the light may not necessarily propagate along a straight line. Now consider a medium with an index of refraction given by $n(x, y, z) = (1 + \alpha z)n_0$, where $\alpha$ and $n_0$ are constants. A narrow beam of light starts in this medium at point $(0, 0, 0)$ with the initial propagation vector along the $y$-direction. Find the function that describes the path of the beam in this medium.

Problem 4. A particle moves in a spiral orbit $r = \alpha \theta$, where $\alpha$ is a constant. If $\theta$ increases linearly with time, is the force a central field? If not, determine how $\theta$ must vary with time for a central force.