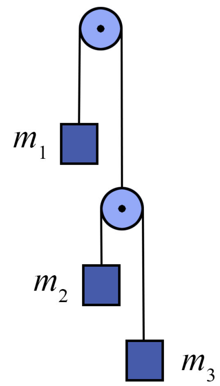
Name: ____________________

PHYS 505 - Classical Mechanics (graduate), Fall 2015
Instructor: Sergiy Bubin
Midterm Exam 1

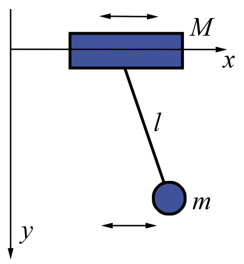
Instructions:

- All problems are worth the same number of points (although some might be more difficult than the others). The problem for which you get the lowest score will be dropped. Hence, even if you do not solve one of the problems you can still get the maximum score for the exam.
- This is a closed book test. No notes, books, phones, tablets, calculators, etc. should be used.
- No communication with classmates is allowed during the test.
- Show your work, explain your reasoning. Final answers or intermediate steps without clear explanations will receive no credit.
- Write legibly. If I cannot read and understand it then I will not be able to grade it.
- Make sure pages are stapled together before submitting your work.

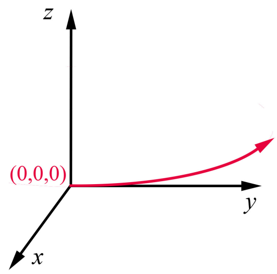
Problem 1. Consider the system of three masses connected with two weightless pulleys as shown in the figure. The top pulley's position is fixed, while the bottom one can move up and down. Write down the Lagrangian for this system, generate Lagrange equation(s), identify any cyclic coordinate(s), and solve the equation(s). What are the accelerations of masses m_1, m_2, and m_3?



Problem 2. A block of mass M is free to slide forth and back in the x-direction on a horizontal bar without friction. A bob of mass m is attached to the bottom of the block with a massless rigid rod of length l. The bob can oscillate freely in the xy-plane. Write down the Lagrangian for this system, generate Lagrange equation(s), identify any cyclic coordinate(s), and solve the equation(s) in the case of small amplitude oscillations.



Problem 3. The Fermat principle states that the light propagates along the path which takes the minimum amount of time. Recall that the local speed of light in a medium with the index of refraction n is $v = c/n$, where c is the speed of light in vacuum. From the Fermat principle it follows that for a medium with a nonuniform index of refraction the light may not necessarily propagate along a straight line. Now consider a medium with an index of refraction given by $n(x, y, z) = (1 + \alpha z)n_0$, where α and n_0 are constants. A narrow beam of light starts in this medium at point $(0, 0, 0)$ with the initial propagation vector along the y-direction. Find the function that describes the path of the beam in this medium.



Problem 4. A particle moves in a spiral orbit $r = \alpha\theta$, where α is a constant. If θ increases linearly with time, is the force a central field? If not, determine how θ must vary with time for a central force.