Instructions:

- All problems are worth the same number of points (although some might be more difficult than the others).
- This is a closed book test. No notes, books, phones, tablets, calculators, etc. should be used.
- No communication with classmates is allowed during the test.
- Show your work, explain your reasoning. Final answers or intermediate steps without clear explanations will receive no credit.
- Write legibly. If I cannot read and understand it then I will not be able to grade it.
- Make sure problems are marked and pages are stapled together before submitting your work.

Problem 1. Calculate the moment of inertia of a homogeneous cone of mass $M$, height $H$, and radius $R$ about an axis that lies on the surface of the cone (and passes through its apex).

Problem 2. A symmetric top ($I_1 = I_2 \neq I_3$) is rotating freely about its center of mass. Then a frictional torque $N_f = -b \omega$ is applied to slow down the rotation, where $b$ is a positive constant. Find the component of the angular velocity along the symmetry axis as a function of time.

Problem 3. Mass $m$ is connected over a pulley to a spring, which is attached to a wall, as depicted in the figure. The force constant of the spring is $k$, while the pulley is a uniform cylinder of mass $M$. What is the Lagrangian of the system? Mass $m$ is pulled down slightly and then released. Find the frequency of small oscillations.

Problem 4. Consider two suspended pendula of equal length $l$ as shown in the figure. The mass of the first pendulum is $2m$, while the second one is $m$. They are connected with a weightless spring of force constant $k$. Find the normal frequencies and normal modes for this system.